Matrices and Linear Algebra

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What is ~~the~~ a matrix?

- ▸ What is the matrix?
- A matrix is a rectangular arrangement of numbers:

$$\begin{bmatrix} 10 & 1 & 9 \\ 15 & 2 & 8 \end{bmatrix}$$

- The above matrix is a 2 x 3 matrix, as it has **2 rows** and **3 columns**.
- Each number in the matrix is called an element. These are *real* numbers

Row and Column Vectors

- A 1 x 3 matrix is a **vector** in 3D, which is written as a row:

$$\begin{bmatrix} 20 & 30 & 40 \end{bmatrix}$$

- A 3 x 1 matrix is a column vector:

$$\begin{bmatrix} 20 \\ 30 \\ 40 \end{bmatrix}$$

- An *n* x *n* matrix is a square matrix.

Notational Conventions

- Matrices denoted by capital letters e.g A, B...
- The elements of a matrix are denoted as a_{ij}.
  - ith row and jth column.

$$A_{m,n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix}$$

Java Notations

- In Java, a matrix can be represented by a two-dimensional array.
- `int a [][] = new int[m][n];`
- An element of the array can be selected using `a[i][j]`

Matrix Addition

- Matrices are compared componentwise - having similar dimensions.
- Two matrices of the same dimensions can be added componentwise:

$$\begin{bmatrix} 1 & 2 & 3 \\ -2 & -1 & 5 \\ 1 & 4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 0 \\ 5 & 3 & -2 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+1 & 2+2 & 3+0 \\ -2+5 & -1+3 & 5+(-2) \\ 1+0 & 4+2 & 1+1 \end{bmatrix}$$

- Subtraction works analogously.

Scaling a matrix

- A matrix can be scaled by any factor s:
- $5.\begin{bmatrix} 1 & 2 & 3 \\ -2 & -1 & 5 \\ 1 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 10 & 15 \\ -10 & -5 & 25 \\ 5 & 20 & 5 \end{bmatrix}$

Laws for Addition and Scalar Multiplication

- $A + B = B + A$
- $A + (B + C) = (A + B) + C$
- $A + 0 = 0 + A$
- $1.A = A$
- $0.A = 0$
- $(s + t).A = s.A + t.A$
- $s.(A + B) = s.A + s.B$
- $(s \times t).A = s.(t.A)$

Matrix Multiplication

- Essential part of graphical manipulation in a 3D coordinate system.
- Involves mixing rows and columns.
- $$\begin{bmatrix} 4 & 6 \\ -3 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 4 \times 1 + 6 \times -1 & 4 \times 3 + 6 \times 0 \\ -3 \times 1 + 1 \times -1 & -3 \times 3 + 1 \times 0 \end{bmatrix} = \begin{bmatrix} -2 & 12 \\ -4 & -9 \end{bmatrix}$$

AB $\neq BA$!!!

- Matrix multiplication is not commutative.
- $$\times \begin{bmatrix} 4 & 6 \\ -3 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \times 4 + 3 \times -3 & 1 \times 6 + 3 \times 1 \\ -1 \times 4 + 0 \times -3 & -1 \times 6 + 0 \times 10 \end{bmatrix} = \begin{bmatrix} -5 & 9 \\ -4 & -6 \end{bmatrix}$$

Identity Matrix

- Important in graphics processing tasks.
- $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- $AE = EA = A$

Laws of Matrix Multiplication

- $ABC = A(BC)$
- $A(B + C) = AB + AC$
- $(A + B)C = AC + BC$
- $A0 = 0A = 0$
- $s.(AB) = s.AB = A(s.B)$

Exercises

Exercises from Tutorial Notes